

Algebra 2 — Prep & Practice Guide

Topic: Quadratic functions in vertex form

How to use this guide. If this material is new to your child, read Part A together first, then do Part B. If it has already been taught, try Part B first — and use Part A afterwards for whatever was missed. Either way, upload the finished Part B pages as a Performance Review with this guide as the prior guide: the answers come back with that review, and it ends with the next practice set.

Part A — Learn

6 short lessons, one for each idea in the practice test that follows. Each says what the idea is in the course's own method, works one example with the reasoning before the answer, and names the mistake to watch for. Each lesson also has a two-minute video, on this guide's page in your Guru portal.

Identifying the vertex, axis of symmetry, and max/min value from vertex form (watching the sign of h)

Vertex form is $f(x) = a(x - h)^2 + k$, and the vertex is always the point (h, k) . The axis of symmetry is the vertical line $x = h$, and the whole graph mirrors across that line. If $a > 0$ the parabola opens up and k is the minimum value; if $a < 0$ it opens down and k is the maximum value. This idea applies any time you're handed an equation already in vertex form and asked to describe the graph without graphing it. The tricky part is always reading h correctly, since the form has a minus sign built in.

WORKED EXAMPLE

For $f(x) = -(x + 2)^2 + 5$, give the vertex, the axis of symmetry, the direction the graph opens, and the maximum or minimum value.

First rewrite $x + 2$ as $x - (-2)$ so it matches the form $x - h$, which shows $h = -2$, not 2. The vertex is $(h, k) = (-2, 5)$. The axis of symmetry is the vertical line through the vertex, $x = -2$. Next look at a : here $a = -1$, which is negative, so the parabola opens downward. Since it opens down, the vertex is the highest point, so $k = 5$ is the maximum value, not a minimum.

Answer: Vertex $(-2, 5)$; axis of symmetry $x = -2$; opens downward; maximum value 5.

WATCH OUT — Students see $(x + 2)$ and read $h = 2$ instead of $h = -2$, flipping the sign. Catch this by always rewriting the expression as $x - (\text{something})$ first, and double-check by plugging $x = h$ back in to confirm it gives k .

Describing transformations of $y = x^2$ from the values of a , h , and k

Every piece of $a(x - h)^2 + k$ moves or reshapes the basic parabola $y = x^2$ in its own way. The value of a flips the graph upside down if it's negative, and stretches the graph narrower if $|a| > 1$ or shrinks it wider if $0 < |a| < 1$. The value of h slides the graph left or right — right if h is positive, left if h is negative, since it's a

shift in the x-direction. The value of k slides the graph straight up or down by that many units. This applies whenever you're asked to describe a graph in words rather than just locate its vertex.

WORKED EXAMPLE

Describe how the graph of $y = -3(x - 1)^2 + 4$ is obtained from the graph of $y = x^2$.

Start with $a = -3$. The negative sign means the graph reflects across the x-axis, so it opens downward instead of upward. The size, 3, is greater than 1, so the graph is also stretched vertically, making it narrower than $y = x^2$. Next look at h : the expression is $(x - 1)$, so $h = 1$, meaning the graph shifts right 1 unit. Finally $k = 4$ means the graph shifts up 4 units. Put all of these together in order: reflect, stretch, then shift.

Answer: The graph of $y = x^2$ is reflected across the x-axis, stretched vertically by a factor of 3, shifted right 1 unit, and shifted up 4 units.

WATCH OUT — Students often forget to mention the reflection when a is negative, focusing only on the stretch/shrink size. Notice this by checking the sign of a separately from its size before writing the description.

Finding y-intercept and x-intercepts of a quadratic given in vertex form

To find the y-intercept of a quadratic in vertex form, plug in $x = 0$ and simplify — this works because the y-intercept is just the function's value when $x = 0$. To find the x-intercepts, set $f(x) = 0$ and solve by isolating the squared term and taking square roots, remembering the $\pm$ sign. This applies whenever a quadratic is already in vertex form and you need key points for graphing or for answering a question about where the graph crosses the axes. Not every parabola has x-intercepts, so it's normal sometimes to find none.

WORKED EXAMPLE

For $g(x) = \frac{1}{2}(x - 4)^2 - 2$, find the vertex, the y-intercept, and the x-intercepts.

The vertex comes straight from the form: $h = 4$, $k = -2$, so vertex $(4, -2)$. For the y-intercept, substitute $x = 0$: $g(0) = \frac{1}{2}(0-4)^2 - 2 = \frac{1}{2}(16) - 2 = 8 - 2 = 6$, giving the point $(0, 6)$. For the x-intercepts, set $g(x) = 0$: $\frac{1}{2}(x-4)^2 - 2 = 0$, so $\frac{1}{2}(x-4)^2 = 2$, then $(x-4)^2 = 4$. Taking square roots of both sides gives $x - 4 = \pm 2$, so $x = 6$ or $x = 2$.

Answer: Vertex $(4, -2)$; y-intercept $(0, 6)$; x-intercepts $(2, 0)$ and $(6, 0)$.

WATCH OUT — Students often take the square root too early, before fully isolating $(x - h)^2$, or they forget the $\pm$ and only get one x-intercept. Check by making sure the squared term is completely alone on one side before applying the square root, and always write both the $+$ and $-$ cases.

Converting standard form to vertex form by completing the square (including factoring out a when $a \neq 1$)

Completing the square turns $y = ax^2 + bx + c$ into vertex form by building a perfect square trinomial out of the x-terms. When $a = 1$, take half of b , square it, and add and subtract that number. When $a \neq 1$, first factor a out of the x^2 and x terms before completing the square inside the parentheses, being careful with the leftover constant outside. This applies whenever a quadratic is given in standard form but a question asks

for the vertex, intercepts, or a description of transformations, since those all need vertex form. As a shortcut check, the vertex's x-coordinate should always equal $-b/(2a)$.

WORKED EXAMPLE

Write $y = 3x^2 + 12x + 7$ in vertex form. State the vertex.

Since $a = 3$ is not 1, factor 3 out of the first two terms only: $y = 3(x^2 + 4x) + 7$. Inside the parentheses, take half of 4, which is 2, and square it to get 4. Add and subtract 4 inside the parentheses: $3(x^2 + 4x + 4 - 4) + 7$. Pull the -4 out, but remember it's multiplied by the 3 outside, so it becomes -12 when it leaves the parentheses: $3(x+2)^2 - 12 + 7$. Combine the constants: $-12 + 7 = -5$. Check with the shortcut: $-b/(2a) = -12/6 = -2$, which matches $h = -2$.

Answer: $y = 3(x + 2)^2 - 5$; vertex $(-2, -5)$.

WATCH OUT — The most common error is adding and subtracting the squared number before factoring out a , so the number that leaves the parentheses doesn't get multiplied by a . Check by using the shortcut $x = -b/(2a)$ to verify h , and plugging $x = h$ into the original standard form to verify k .

Solving a quadratic equation in vertex form using square roots, including recognizing no real solutions

When a quadratic equation is already in vertex form and set equal to zero (or any number), isolate the squared expression $(x - h)^2$ first, then take the square root of both sides, remembering to write both the positive and negative root. This applies to any equation of the form $a(x-h)^2 + k = (\text{number})$. If isolating the squared term leads to it equaling a negative number, there is no real solution, because a real number squared can never be negative. Recognizing this early saves time instead of trying to force an answer.

WORKED EXAMPLE

Solve $2(x + 3)^2 - 18 = 0$.

Add 18 to both sides to isolate the term with the square: $2(x+3)^2 = 18$. Divide both sides by 2: $(x+3)^2 = 9$. Since the squared expression is isolated and equals a positive number, take the square root of both sides, keeping both signs: $x + 3 = \pm 3$. Split into two cases: $x + 3 = 3$ gives $x = 0$, and $x + 3 = -3$ gives $x = -6$.

Answer: $x = 0$ or $x = -6$.

WATCH OUT — Students often take the square root while a constant is still attached outside, or they drop the $\pm$ and report only one solution. Check by confirming $(x-h)^2$ is completely alone before rooting, and always write the $\pm$ step explicitly even if one answer later gets rejected.

Writing a quadratic equation in vertex form from a given vertex and one other point, and applying it to a real-world max-height/time problem

If you know the vertex (h, k) of a parabola and one other point on the graph, you can find a by substituting both into vertex form and solving for the only unknown. Plug the vertex directly in for h and k , then substitute the other point's x and y to get an equation with just a in it. This is especially useful in real-world problems like projectile motion, where the vertex often represents the maximum height and the time it occurs, and the

equation lets you answer follow-up questions like when the object hits the ground.

WORKED EXAMPLE

A parabola has its vertex at (2, -3) and passes through (4, 5). Write its equation in vertex form.

Start with the general vertex form $y = a(x - h)^2 + k$ and substitute $h = 2$ and $k = -3$ since those come from the vertex: $y = a(x - 2)^2 - 3$. Now use the other point (4, 5) by substituting $x = 4$ and $y = 5$: $5 = a(4-2)^2 - 3$. Simplify inside the parentheses: $5 = a(2)^2 - 3$, so $5 = 4a - 3$. Add 3 to both sides: $8 = 4a$. Divide by 4: $a = 2$.

Answer: $y = 2(x - 2)^2 - 3$.

WATCH OUT — Students sometimes substitute the vertex and the other point backwards, plugging the extra point's coordinates in for h and k instead of x and y . Check by confirming that the values used for h and k match the vertex given in the problem, and that the remaining point is only used for x and y .

Part B — Practice. A practice test on the ideas these pages say matter most at this level. Level: Algebra 2, Unit 4 (Quadratic Functions), Lesson 4.3: vertex form of a quadratic.

12 questions across 6 ideas these pages cover — two for each, in the form the course uses. Work them on these pages. **The answers are not printed here: they come back with the review.** When the pages are done, upload them as a Performance Review with this guide as the prior guide, and Guru grades them and says which gaps have closed.

Identifying the vertex, axis of symmetry, and max/min value from vertex form (watching the sign of h)

1. For $f(x) = -(x - 3)^2 + 7$, give the vertex, the axis of symmetry, the direction the graph opens, and the maximum or minimum value.

Answer: _____

2. For $f(x) = 2(x + 4)^2 - 6$, give the vertex, the axis of symmetry, the direction the graph opens, and the maximum or minimum value.

Answer: _____

Describing transformations of $y = x^2$ from the values of a, h, and k

3. Describe how the graph of $y = 2(x + 3)^2 - 5$ is obtained from the graph of $y = x^2$.

Answer: _____

4. Describe how the graph of $y = -\frac{1}{4}(x - 5)^2 + 2$ is obtained from the graph of $y = x^2$.

Answer: _____

Finding y-intercept and x-intercepts of a quadratic given in vertex form

5. For $g(x) = \frac{1}{3}(x - 6)^2 - 3$, find the vertex, the y-intercept, and the x-intercepts.

Answer: _____

6. For $g(x) = -2(x + 1)^2 + 8$, find the vertex, the y-intercept, and the x-intercepts.

Answer: _____

Converting standard form to vertex form by completing the square (including factoring out a when $a \neq 1$)

7. Write $y = x^2 - 10x + 18$ in vertex form. State the vertex.

Answer: _____

8. Write $y = 2x^2 - 8x + 3$ in vertex form. State the vertex.

Answer: _____

Solving a quadratic equation in vertex form using square roots, including recognizing no real solutions

9. Solve $3(x - 2)^2 - 12 = 0$.

Answer: _____

10. Solve $(x + 4)^2 + 5 = 0$, or explain why it has no real solutions.

Answer: _____

Writing a quadratic equation in vertex form from a given vertex and one other point, and applying it to a real-world max-height/time problem

11. A parabola has its vertex at $(1, -4)$ and passes through $(3, 4)$. Write its equation in vertex form.

Answer: _____

12. A rocket's height in feet after t seconds is modeled by $h(t) = -16(t - 2)^2 + 68$. (a) What is the greatest height the rocket reaches, and at what time? (b) After how many seconds does the rocket hit the ground? Round to the nearest hundredth.

Answer: _____

Built from: the ideas these pages say matter most at this level. Guru Seal: every question and its kept answer key were independently checked by separate AI systems; 0 item(s) were dropped in that check.